A GAME-THEORETIC MODEL FOR “ZERO-INTEREST” INSTALMENT SCHEMES

Mihir Dash, Professor  
School of Business  
Alliance University, India  
Teja Motukuri, MBA, Relationship Manager  
ICICI Bank, India

Abstract
The present study extends the previous analysis by proposing a game-theoretic model for the optimal choice of ‘zero-interest’ instalment scheme. The choice of scheme is analysed in two stages: the extent of down-payment the consumer is willing to pay, and the payment period that the financing institution is willing to offer. The study suggests that, by restructuring loan repayment terms, it may sometimes be possible to benefit both financial institutions and consumers. Further extension of the present study would be possible using multi-criteria decision methods, as individual consumers need to balance between decision criteria including the amount of down-payment that they have to immediately pay, the period over which they can spread the payments, and the overall net present value of the scheme in their choice of scheme.

Keywords: ‘zero-interest’ instalment scheme, game-theoretic model, multi-criteria decision methods

JEL Codes: C70, G11, G21

Received: January 20, 2019  
Revision: February 8, 2019  
Accepted: February 11, 2019

Introduction
‘Zero-interest’ instalment schemes have become a popular tool for retailers to make their products more affordable to consumers. The trend is growing fastest for the consumer durables segment, where products tend to be highly-priced and not easily affordable for households with low/medium income (Sengupta, 2003; Maji, 2016; Lee, 2017). Dash and Motukuri (2018) had proposed a model for evaluating the net present value of ‘zero-interest’ instalment schemes. This study analyses ‘zero-interest’ instalment schemes in a game-theoretic framework.

There are three distinct groups of players involved in the transaction: the retailers, the consumers, and the financing institutions. However, the role of retailers is limited, so the problem can be modelled as a two-player game. As observed in Dash and Motukuri (2018), the choice of scheme can be analysed in two stages: the extent of down-payment the consumer is willing to pay, and the payment period that the financing institution is willing to offer. The consumers would want to choose a scheme with low percentage down-payment and long payment period, while the financing institutions would want consumers to choose a scheme with high percentage down-payment and short payment period. The study extends the models of net present value for the financing institutions and for the consumers discussed in Dash and Motukuri (2018).
Net Present Value Models for Financing Institutions and for Consumers

Assumptions
1. The original/list price of the product is $P$.
2. The customer is required to make a down-payment of $\phi P$ upon purchase of the product, and instalments of $(1-\phi)P/n$ for $n$ months, starting from the end of the first month onwards. This is referred to as an “$\phi/n$ scheme”.
3. The financing institution receives a commission of $\theta P$ from the retailer at the time of the transaction.
4. The opportunity cost for the financing institution is $r_f$ and for the consumer is $r_c$.

The cash flows for the financing institution are modelled as follows:
1. The initial cash outflow is equal to $-(1-\phi)P + \theta P$.
2. The first month cash inflow (at the end of the month) is $(1-\phi)P/n$.
3. The cash inflows for the next $n-1$ months (starting from the end of the second month) are all equal to $(1-\phi)P/n$.

The net present value ($NPV_f$) of the above cash flows, assuming a rate of return of $r$, is given by:

$$NPV_f(r) = -(1-\phi-\theta)P + \frac{(1-\phi)P}{n(1+r/12)} + \frac{(1-\phi)P}{n(1+r/12)^2} + \ldots \frac{(1-\phi)P}{n(1+r/12)^n}.$$

The financing institutions would want to offer a scheme which is viable for them, i.e. if the $NPV_f$ is positive (Brealey and Myers, 2016; Yankovoy, 2012; Yankovoy and Melnik, 2012; Goncharuk, 2012; Goncharuk, 2015). It can be observed that $NPV_f$ decreases with increasing $n$ for each fixed $\phi$ and increases with $\phi$ for each fixed $n$.

The cash flows for the consumer are modelled as follows:
1. The initial cash outflow is equal to $\phi P$.
2. The cash outflows for the next $n$ months (starting from the end of first month) are all equal to $(1-\phi)P/n$.

The net present value ($NPV_c$) of the above cash flows, assuming an opportunity cost of $r$, is given by:

$$NPV_c(r) = \phi P + \frac{(1-\phi)P}{n(1+r/12)} + \frac{(1-\phi)P}{n(1+r/12)^2} + \ldots \frac{(1-\phi)P}{n(1+r/12)^n}.$$

The consumers would want to choose a scheme for which the $NPV_c$ is minimum among the schemes available to them. It can be observed that $NPV_c$ decreases with increasing $n$ for each fixed $\phi$ and increases with $\phi$ for each fixed $n$.

The problem may be formulated as a two-player game as follows:
I. The consumer selects a down-payment percentage that they are willing to pay at present. The financing institution then offers different viable payment periods based on this down-payment percentage, and the consumer selects the payment period for
which $NPV_c$ is minimum. Mathematically, this may be formulated as:

$$\begin{align*}
    \text{min} & \quad NPV_c(\phi, n) \\
    \text{s.t.} & \quad NPV_f(\phi, n) \geq 0 \\
    & \quad 0 \leq \phi \leq 1 \text{ fixed} \\
    & \quad n = 6, 7, \ldots, 24
\end{align*}$$

II. The consumer selects a payment period which they are willing to spread the payment over. The financing institution then specifies the viable down-payment percentages based on this payment period, and the consumer selects the down-payment percentage for which $NPV_c$ is minimum. Mathematically, this may be formulated as:

$$\begin{align*}
    \text{min} & \quad NPV_c(\phi, n) \\
    \text{s.t.} & \quad NPV_f(\phi, n) \geq 0 \\
    & \quad n \text{ fixed} \\
    & \quad 0 \leq \phi \leq 1
\end{align*}$$

**Application**

The schemes offered by the financial institutions range from six to twelve months, with down-payment percentages ranging from 10% to 50%. The commission offered by the dealer to the financial institutions is given by $\theta = 2.25\%$. The analysis is performed for an item with a unit cost of Rs. 100,000. The NPVs for the financial institutions at 8% rate of return and the NPVs for the consumers at 12% opportunity cost under the different schemes are presented in Table 1.

**Table 1. NPVs of different schemes at 8% rate of return for financing institutions and 12% opportunity cost for consumers**

<table>
<thead>
<tr>
<th></th>
<th>10%</th>
<th>20%</th>
<th>30%</th>
<th>40%</th>
<th>50%</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>186.78, 96,932.15</td>
<td>(416.03, 97,273.02)</td>
<td>(645.27, 97,613.89)</td>
<td>(874.52, 97,954.76)</td>
<td>(1103.77, 98,295.64)</td>
</tr>
<tr>
<td>7</td>
<td>(-102.79, 96,505.36)</td>
<td>(158.63, 96,893.65)</td>
<td>(420.05, 97,281.95)</td>
<td>(681.47, 97,670.24)</td>
<td>(942.90, 98,058.53)</td>
</tr>
<tr>
<td>8</td>
<td>(-391.08, 96,081.37)</td>
<td>(-97.63, 96,516.78)</td>
<td>(195.82, 96,952.18)</td>
<td>(489.28, 97,387.58)</td>
<td>(782.73, 97,822.99)</td>
</tr>
<tr>
<td>9</td>
<td>(-678.11, 95,660.18)</td>
<td>(-352.76, 96,142.38)</td>
<td>(-27.42, 96,624.58)</td>
<td>(297.93, 97,106.78)</td>
<td>(623.27, 97,588.99)</td>
</tr>
<tr>
<td>10</td>
<td>(-963.87, 95,241.74)</td>
<td>(-606.77, 95,770.44)</td>
<td>(-249.68, 96,299.13)</td>
<td>(107.42, 96,827.83)</td>
<td>(464.52, 97,356.52)</td>
</tr>
<tr>
<td>11</td>
<td>(-1248.38, 94,826.05)</td>
<td>(-859.67, 95,400.93)</td>
<td>(-470.96, 95,975.82)</td>
<td>(-82.25, 96,550.70)</td>
<td>(306.46, 97,125.58)</td>
</tr>
<tr>
<td>12</td>
<td>(-1531.64, 94,413.08)</td>
<td>(-1111.45, 95,033.85)</td>
<td>(-691.27, 95,654.62)</td>
<td>(-271.09, 96,275.39)</td>
<td>(149.09, 96,896.16)</td>
</tr>
</tbody>
</table>

Thus, the optimal scheme for the consumers in the above table is found to be the 40%/10 scheme, yielding the financing institution $NPV_f = \text{Rs. 107.42}$ and minimising the $NPV_c = \text{Rs. 96,827.83}$. The global minimum, based on the non-linear programming problem of minimising $NPV_c(\phi, n)$ subject to $NPV_f(\phi, n) \geq 0$, with $0 \leq \phi \leq 1$, and $n = 6, 7, \ldots, 24$, is found at $n = 6$, $\phi = 1.8524\%$, and $NPV_c(\phi, n) = \text{Rs. 96,654.42}$, with $NPV_f(\phi, n) = \text{Rs. 0.00}$. Subsequently introducing a minimum down-payment constraint $\phi \geq 10\%, 20\%, 30\%, 40\%$, and $50\%$ yielded solutions respectively: $n = 7$, $\phi = 13.9319\%$, and $NPV_c(\phi, n) = \text{Rs. 96,658.03}$; $n = 8$, $\phi = 23.3269\%$, and $NPV_c(\phi, n) = \text{Rs. 96,661.61}$; $n = 9$, $\phi = 30.8427\%$, and $NPV_c(\phi, n) = \text{Rs. 96,665.22}$; $n = 11$, $\phi = 42.1160\%$, and
\[ NPV_c(\phi, n) = \text{Rs. 96,672.35}; \text{ and } n = 12, \phi = 50.1680\%, \text{ and } NPV_c(\phi, n) = \text{Rs. 96,679.42}, \text{ with } NPV_f(\phi, n) = \text{Rs. 0.00} \text{ in all cases. A similar constraint can be introduced for the payment period, viz. } n \geq 6, 7, \ldots, 24, \text{ with similar results.} \]

**Discussion**

The study extends the analysis of n/k-schemes examined in Dash and Motukuri (2018), in which customers simultaneously select the payment period and the down-payment amount (in multiples of the monthly payment). The optimal n/k-scheme was found to be the 11/3-scheme, which resulted in net present values \( NPV_f = \text{Rs. 100.89} \) for the financing institutions and \( NPV_c = \text{Rs. 96,833.43} \) for the consumers.

The study extends Dash and Motukuri (2018) by allowing consumers to first select the down-payment percentage, and then accordingly selecting the payment period, subject to viability for the financing institutions. The comparable optimal \( \phi/n \)-schemes obtained in current study are the 23.3269%/8-scheme and the 30.8427%/9-scheme, both of which result in lower net present value for the consumers, viz. \( NPV_c = \text{Rs. 96,661.61} \) and \( NPV_c = \text{Rs. 96,665.22} \). Thus, the \( \phi/n \)-schemes may be considered more beneficial to consumers than the n/k-schemes, though at an opportunity cost to the financial institutions. On the other hand, the optimal \( \phi/n \)-scheme identified, viz. the 40%/10-scheme, which resulted in net present values \( NPV_f = \text{Rs. 107.42} \) for the financing institutions and \( NPV_c = \text{Rs. 96,827.83} \) for the consumers, is better than the optimal n/k-scheme, for both financing institutions and consumers. Thus, both the financing institutions and the consumers are benefitted by the \( \phi/n \)-schemes.

The study suggests that, by restructuring loan repayment terms, it may sometimes be possible to benefit both financial institutions and consumers. The extent to which the restructuring can benefit financial institutions and consumers depends on the down-payment consumers can afford to pay and the payment period that they are comfortable with. Financial planners should experiment with different loan repayment terms to identify those which best suit the consumers’ requirements while being viable for both the financing institutions and the consumers.

A possible extension of the present study would be to analyse the problem using multi-criteria decision methods. As observed in Dash and Motukuri (2018), individual consumers may need to balance between several decision criteria, including the amount of down-payment that they have to immediately pay, the period over which they can spread the payments, and the overall net present value of the scheme. Thus, multi-criteria decision methods may be applicable.

**References**


